

Lösungen Klasse 5 (Grundlagenfach)

Rechenregeln mit dem Summenzeichen

$$\begin{aligned} 1. \sum_{i=1}^n (3i-1)^2 &= \sum_{i=1}^n (9i^2 - 6i + 1) = 9 \sum_{i=1}^n i^2 - 6 \sum_{i=1}^n i + \sum_{i=1}^n 1 = 9 \frac{n(n+1)(2n+1)}{6} - 6 \frac{n(n+1)}{2} + n = \\ &= \frac{n}{2} (6n^2 + 9n + 3 - 6n - 6 + 2) = \underline{\underline{\frac{n}{2} (6n^2 + 3n - 1)}} \end{aligned}$$

$$\text{z.B. für } n = 4: \sum_{i=1}^4 (3i-1)^2 = 2^2 + 5^2 + 8^2 + 11^2 = 214 = 2(6 \cdot 16 + 12 - 1)$$

$$\begin{aligned} 2. \sum_{i=1}^n (4i-n)^2 &= \sum_{i=1}^n (16i^2 - 8i \cdot n + n^2) = 16 \sum_{i=1}^n i^2 - 8n \sum_{i=1}^n i + n^2 \sum_{i=1}^n 1 = \\ &= 16 \frac{n(n+1)(2n+1)}{6} - 8n \frac{n(n+1)}{2} + n^2 \cdot n = \frac{n}{3} (16n^2 + 24n + 8 - 12n^2 - 12n + 3n^2) = \\ &= \underline{\underline{\frac{n}{3} (7n^2 + 12n + 8)}} \end{aligned}$$

$$\text{z.B. für } n = 3: \sum_{i=1}^3 (4i-3)^2 = 1^2 + 5^2 + 9^2 = 107 = 7 \cdot 9 + 36 + 8$$

$$\begin{aligned} 3. \sum_{i=1}^{n+1} (4i^2 - 3i) &= 4 \sum_{i=1}^{n+1} i^2 - 3 \sum_{i=1}^{n+1} i = 4 \frac{(n+1)(n+2)(2(n+1)+1)}{6} - 3 \frac{(n+1)(n+2)}{2} = \\ &= \frac{(n+1)(n+2)}{6} (8n + 12 - 9) = \underline{\underline{\frac{(n+1)(n+2)(8n+3)}{6}}} \end{aligned}$$

$$\text{z.B. für } n = 2: \sum_{i=1}^3 (4i^2 - 3i) = 4 - 3 + 16 - 6 + 36 - 9 = 38 = \frac{3 \cdot 4 \cdot 19}{6}$$